Balance sheet policies and Central Bank losses in a HANK model

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Abstract

What are the effects of central bank balance sheet expansion, and should we worry about central bank losses? Using a Heterogeneous-Agent New Keynesian model incorporating money in utility and an endogenous zero lower bound (ZLB), we study the fiscal-monetary interactions of central bank balance sheet policies. We find that the overall efficiency of asset purchase programs depends on the combination of the expected future size of the balance sheet and the fiscal transmission of central bank losses. First, permanent balance sheet expansions stimulate the economy in the long run and by anticipation, increase inflation and output during the ZLB episode, as they interact with distortionary taxes and imperfect capital markets. Second, upon exiting the ZLB, the central bank incurs losses; issuing securities to offset these losses is more welfare-enhancing than raising taxes.

JEL classification – E41, E51, E52, E58, E63, E65

Keywords – monetary policy; heterogeneous agents; balance sheet; Quantitative Easing; Quantitative Tightening; Central Bank losses

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Introduction

Recent central bank balance sheet policies can be divided into three phases: expansion during the Zero Lower Bound (ZLB), potential losses incurred upon exiting the ZLB, and the long-run effects contingent on the chosen normalization policy. This paper analyzes these phases to understand the overall effect of large-scale asset purchase programs implemented by central banks over the last decade. The first phase involved a substantial increase in the size of central bank balance sheets. For instance, the ECB saw its balance sheet swell from 200 to 5,000 billion euros between 2015 and 2023, while the Fed experienced growth from 900 to 9,000 billion dollars from 2008 to 2022.[1] The second phase entails central bank losses resulting from policy rate hikes, ranging from 0.5% to 1.5% of GDP in 2023 for major central banks. These losses come from the interest rate mismatch between low-yielding long-term bonds bought during Quantitative Easing (QE) programs and bank reserves remunerated at the policy rate. The final phase is the convergence toward a new steady state, in which the central bank either returns to its pre-expansion balance sheet size or maintains a permanently larger asset portfolio in real terms.

The economic theory behind each phase remains unclear. In the long run, monetary policy is expected to be neutral, as nominal rigidities are transitory, and the size and composition of the central bank's balance sheet are deemed irrelevant, a result known as Wallace (1981) neutrality. In the short run, during ZLB episodes, balance sheet policies are also expected to be neutral, as the central bank acquires assets with no return, such as public debt, while creating money, which also yields no return, thereby keeping agents' wealth and income levels unchanged.

In this paper, we challenge these neutrality results by examining the role of fiscal distortions and imperfect capital markets. We argue, following Auerbach and Obstfeld (2005), that central bank balance sheet expansions are not neutral, either in the steady state or during ZLB periods, as they give rise to nontrivial fiscal-monetary interactions.

The intuition is the following. Suppose the central bank purchases assets and holds them constant in real terms forever. On the one hand, the central bank earns greater portfolio gains, thereby increasing remittance to the Treasury, which enables the government to lower distortionary taxes on households. On the other hand, since households have fewer assets to self-insure, the interest rate declines, thereby alleviating the Treasury's debt burden and, once again, reducing distortionary taxes. Consequently, the size of the central bank's balance sheet is non-neutral due to the fiscal and monetary interactions involved. Through anticipation effects, this non-neutrality is transmitted

[1] This corresponds to an increase from 2% to 40% of Euro Area GDP, and from 6% to 35% of US GDP.

to the short run, stimulating the economy even during the ZLB period. However, when we exit the ZLB and the interest rate is positive again, money demand falls as money becomes dominated in return, and the central bank makes losses. These losses are offset either through asset sales, security issuance, or transfers from the Treasury. These different actions modify the time path of distortionary taxes, implying that balance sheet policy is not neutral.

This paper presents a Heterogeneous-Agent New Keynesian (HANK) model to analyze and quantify the overall effect of central bank balance sheet policies, considering the expansion, loss, and normalization phases. Households can invest in money, which enters the utility function up to a satiation level, and in public debt, subject to a borrowing constraint. They face idiosyncratic productivity shocks and are subject to distortionary labor taxes. Firms operate under monopolistic competition and face costly price adjustment, which gives rise to the Phillips curve. The government finances debt repayment and public spending through distortionary labor taxes and remittances from the central bank.

The distinguishing feature of the model is the implementation of balance sheet policies. Outside the ZLB, the central bank conducts conventional monetary policy through a Taylor rule, and may also sell its assets (Quantitative Tightening). The quantity of money is determined by the intersection of the money demand curve and the horizontal money supply, as the central bank adjusts supply to match demand at the chosen interest rate. However, at the ZLB, the central bank switches to a money rule: it issues money to purchase public debt, conducting Quantitative Easing, as observed in practice. As agents are indifferent between money and public debt, their portfolio allocation and behavior remain unaffected. Nonetheless, when the economy exits the ZLB and money becomes return-dominated again, they sell their excess money, resulting in a decline in seigniorage revenue[2] and a loss for the central bank. To offset this loss, the central bank may issue securities, sells assets, or receives a transfer from the Treasury. Our balance sheet policy experiments yield two main results.

First, **permanent balance sheet expansions stimulate the economy**, both in the long run and during ZLB episodes through expectations. Following a balance sheet expansion, the central bank earns higher profits which, when transferred to the government, allow for a reduction in distortionary labor taxes, increasing households' welfare. Moreover, with the Central Bank buying public debt, the expansion raises bond prices, or equivalently decreases the interest rate, further alleviating the state's debt

[2]Seigniorage income is typically defined as the difference between interest earned on central bank assets and the costs of issuing currency. In our model, it includes both the inflation tax and the returns on central bank asset holdings (see Section 2.4).

burden. In our benchmark scenario, where the central bank balance sheet increases from 10 to 20% of GDP and then stabilizes at 15%, we find that consumption increases by 0.8% on impact, and by 0.05% in the long run. In the short run, this increase is driven by the decrease in the interest rate through the Euler equation. In the long run, while this decrease in return is detrimental to households, it is outweighed by the benefits derived from the reduction in tax rates. Furthermore, we decompose welfare effects over the transition path and find that limited self-insurance dominates the welfare gains from lower taxes, resulting in an overall welfare decline. This decline is more pronounced for high-income households, who derive a larger share of their income from capital. In contrast, low-income households benefit from lower taxes and increase their consumption.

Second, **issuing CB securities to offset central bank losses is more welfare-enhancing than raising taxes**. In our framework, upon exiting the ZLB, the central bank abandons the direct control of money supply, triggering a drop in money level and then in seigniorage revenue. The central bank can pay for these losses, either by selling its assets, or by receiving a transfer from the Treasury, financed by a tax increase, or it can postpone the payment of these losses by issuing CB securities.[3] These scenarios alter the time path of taxes and public debt available for households, which have real effects in our non-Ricardian framework. We find that issuing CB securities reduces less welfare than tax increases, as it enhances households' insurance and temporarily decreases the real interest rate. This diminishes public debt repayments and reduces labor income taxes today. When the Treasury and the central bank budget constraints can be consolidated (or equivalently, when the Treasury offsets the loss of the central bank), consumption increases by 0.5% on impact following QE, compared to 0.8% when the central bank addresses the loss by itself and issues securities.

Related literature. Our paper contributes to two strands of literature. First, we contribute to the literature dealing with *unconventional monetary policy*, by analyzing the liquidity channel of Quantitative Easing and proposing a mechanism of fiscal and monetary interaction surrounding central bank losses. Among the transmission channels of QE outlined by Krishnamurthy and Vissing-Jorgensen (2011), we emphasize the liquidity channel, as the reduction in liquidity available to households decreases the interest rates, and the signaling channel, as the credible commitment to increasing the balance sheet target stimulates, by anticipation, the economy even at the ZLB. Moreover, we introduce a fiscal channel of QE, as the fiscal and monetary interaction induces a reduction in distortionary taxes which, by anticipation, alters agents' behavior. This

[3] This is a debt security issued by the central bank and reimbursed using future seigniorage income.

extends the findings of Auerbach and Obstfeld (2005), adding a monetary policy switch at the ZLB. Consequently, our results depart from the neutrality results from Wallace (1981) and Woodford (2012), as we consider the fiscal implications of unconventional monetary policy. Apart from the borrowing constraint, we do not consider financial frictions, another avenue to break the Wallace neutrality (see Chen, Cúrdia and Ferrero (2012) for segmented asset markets, Del Negro, Eggertsson, et al. (2017) for firms liquidity frictions, Gertler and Karadi (2011) for private financial intermediation, and E. Sims and Wu (2021) for leverage constraints). Additionally, given the significance of the fiscal transmission of central bank profits and losses in our findings, we relate to the literature on the "non-consolidation" of the budget constraints of the Treasury and the central bank. C. A. Sims (2004) demonstrated that central bank independence carries fiscal implication, as the systematic remittance of profit to the Treasury creates pressure to avoid losses. The issue of the Treasury support has also been discussed in Del Negro and C. A. Sims (2015), Hall and Reis (2015) and Benigno and Nistico (2020). We follow this literature and consider a benchmark asymmetric scenario where the central bank transfers its profit to the Treasury, but not its losses; in such cases, it issues "deferred assets" or "CB securities", which are repaid over time by retaining future profits. Finally, while our consideration of money in the utility function aligns with the framework of Gali (2020), we depart from it by positing that direct control of the money supply is only done at the ZLB.

Second, we relate to the growing literature *on monetary policy in heterogeneous-agent models*. We adopt the Kaplan, Moll and Violante (2018) framework, adding money in the utility and introducing a balance sheet for the central bank. In this regard, our approach aligns closely with that of Cui and Sterk (2021), who investigate the impact of QE on households, emphasizing differences in the marginal propensity to consume out of liquid and illiquid assets. While we also focus on the household level, we emphasize the fiscal and monetary interaction, and we consider a monetary policy switch at the ZLB, instead of a permanent money rule. Lee (2021) finds that QE program benefits all households, but widens the consumption gap between the top 10% and the rest of the distribution. In contrast, focusing on the fiscal and liquidity channels of QE, we find that it reduces inequalities, as it lowers labor taxes (benefiting poor households) and interest rate (affecting wealthy households negatively), echoing the heterogeneity channel in Auclert (2019). Moreover, the reduction in available public debt following QE pushes middle-class households towards the borrowing constraint, preventing them from smoothing the consumption, a mecanism illustrated in Bilbiie and Ragot (2021). Thus, our findings illustrate the Aiyagari and McGrattan (1998) tradeoff between high liquidity and low taxes. Lastly, we draw inspiration from Auclert et al. (2021) to compute our non-linear transitions, adding a guess on the duration of the ZLB event to account for the switch in monetary policy rules during the liquidity trap.

This paper is organized as follows. Section 1 discusses our two non-neutrality channels. Section 2 presents our Benchmark model. Section 3 details our calibration on Euro Area (EA) data. Section 4 quantifies our two non-neutrality results. Finally, Section 5 concludes.

1 Transmission channels of balance sheet expansion

The fiscal and monetary interaction is key to understand the effect of balance sheet policies, both in the short run and in the long run. On the one hand, the balance sheet expansion increases seigniorage revenue for the central bank due to a larger portfolio in the long-run. Conversely, it induces temporary income losses as the economy exits the ZLB, because as the central bank abandons the control of the money supply to revert to an interest rate rule, the quantity of money in circulation falls, because money becomes strictly dominated by public debt. All these gains and losses from the CB may be transmitted to the Treasury, resulting in a decrease or increase in taxes and public debt levels. Combined with our non-Ricardian structure, these two competing effects create the non-neutrality of balance sheet expansion and therefore break the Wallace neutrality. In this section, we provide intuitions for these two channels.

1.1 Permanent balance sheet expansion: a tale of fiscal distortions

In the absence of distortive taxes, the size of the central bank balance sheet does not matter, which is known as the Wallace (1981) neutrality. The intuition is simple: if the central bank holds more public debt, it earns more asset revenue, which is transferred to the government, lowering existing taxes. At the same time, since households hold less public debt, they earn less asset revenue. The two effects compensate each other and the economy is not affected by the size of the balance sheet.

In this paper, we challenge this neutrality result by introducing three distortions. Firstly, we deviate from Wallace neutrality through the introduction of **distortionary taxes** to fund public debt repayment. Following a balance sheet expansion, the tax rate decreases, reducing the labor distortion in the economy. Secondly, we consider an **incomplete financial market**, wherein the reduction in public debt available to households drives down the interest rate. The interest rate drop reduces capital income but also the debt repayment for the fiscal authority, leading to a further decrease in distortionary labor taxes. Thirdly, the decline in interest rate lowers the opportunity cost of holding money, thereby increasing money demand and the **inflation tax**. This creates an additional seigniorage revenue for the central bank.

To consider these three distortions, let's introduce a static model with representative household and firm, a government and a central bank, describing a steady state. We assume that household consumption C is given by:

$$C = \underbrace{(1-\tau)wN}_{\text{Net labor income}} + \underbrace{rd^H}_{\text{Financial income}} - \underbrace{m\pi}_{\substack{\text{Money return} \\ \text{(Inflation tax)}}}$$

with N the labor supply, d^H the public debt held by households and m real money holdings. We assume an endogenous labor supply given by $N = N(C, \tau)$ and a money demand given by $m = m(C, r)$. The latter rules may be obtained with labor and money in the utility function. Finally, we assume that households' asset demand depends on the interest rate and is given by $d^H = d^H(r)$, which can be obtained with incomplete financial market or an OLG specification. The firm produces according to the production function $Y = N$, giving the first order condition $w = 1$. The government has the following budget constraint:

$$r\bar{d} = \tau w N + s^{CB}$$

The repayment of the constant public debt $\bar{d}$ is financed by taxing labor income at rate τ, and by the profit of the central bank s^{CB}. This profit is given by

$$s^{CB} = m\pi + rd^{CB}$$

The central bank makes profit s^{CB} through its seigniorage revenue $m\pi$, and its asset revenue through its public debt holding d^{CB}. Finally, we have the market clearing condition

$$\bar{d} = d^H + d^{CB}$$

meaning that the total constant public debt must be held either by households or the central bank. Solving for the consumption and differentiating it with respect to a change in d^{CB}, *i.e.* an increase in the size of the central bank balance sheet, we obtain the following formula (proof in Appendix A.1):

$$\frac{\mathrm{d}C}{\mathrm{d}(d^{CB})} = \frac{\overbrace{\left(-\frac{\mathrm{d}r}{\mathrm{d}(d^{CB})}\right) d^H}^{\text{Financial friction}} + \overbrace{\left(-\frac{\mathrm{d}r}{\mathrm{d}(d^{CB})}\right) |\epsilon_{m,r}| m\pi}^{\text{Inflation tax}} + r}{\underbrace{\frac{1-n_C}{|\epsilon_{n,\tau}|}}_{\text{Labor distortion}} - \tau - \pi m_C} \tag{1}$$

with $\epsilon_{y,x}$ the semi-elasticity of y with respect to x, and y_x the partial derivative of y with respect to x. In this formula, we can directly see our three distortions, and how they affect aggregate consumption. For reasonable calibration (see Appendix A.1), the total change of consumption following balance sheet expansion is positive. First, the decrease in public debt available to households decreases the interest rate, lowering the debt burden and then the distortionary labor tax. Second, the change in interest rate increases the money demand, and then the inflation tax, and then decreases the distortionary labor tax. Third, both changes in the distortionary labor tax increase labor supply, increasing consumption.

Then, the first transmission channel of balance sheet expansion in our model relies on the presence of fiscal distortions, allowing us to break the Wallace neutrality. Our quantitative model in section 2 incorporates these three distortions, as well as monopolistic competition and nominal rigidities embedded in the New Keynesian framework.

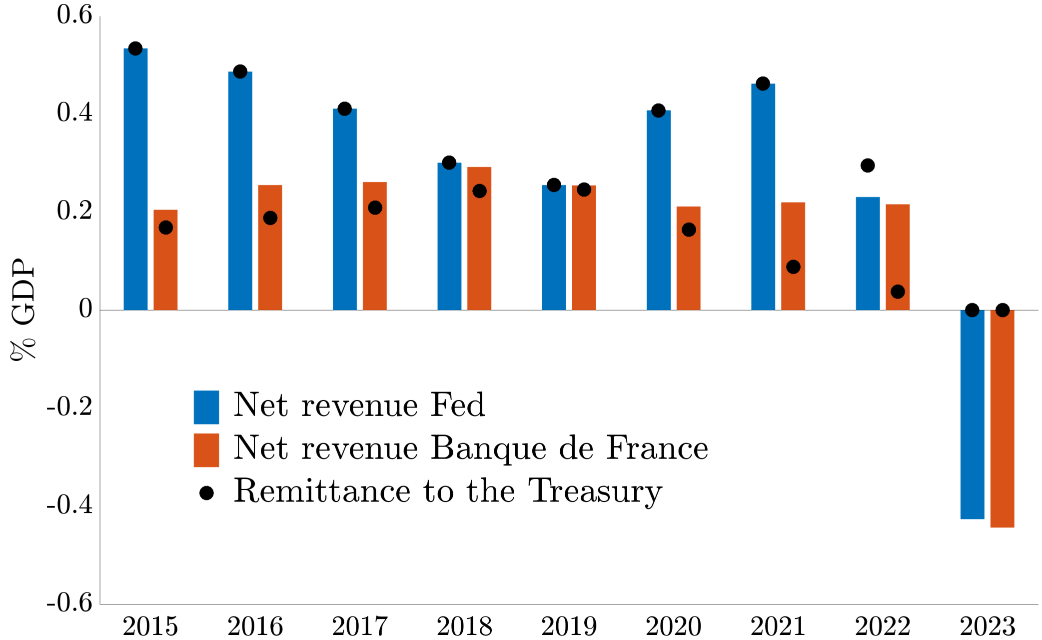
1.2 The fiscal transmission of Central Bank losses

The second avenue through which balance sheet expansion operates is related to how fiscal policy is affected by central bank profits or losses, and how agents react to the future trajectory of taxes and public debt. In this section, we justify why the Treasury and the central bank budget constraints may be separated, and provide examples of losses currently incurred by central banks, and how they are offset.

Central bank assets generally carry a higher interest rate than central bank liabilities, and this spread usually generates positive income, as explained in Long and Fisher (2024). First, by issuing currency or unremunerated reserves for prudential purposes, and backing these liabilities with interest-bearing assets, the central bank generates seigniorage revenue. This revenue can diminish if the interest rate falls to zero or become negative if money demand collapses. Second, by purchasing assets through liquidity emissions at a lower cost than the asset return, the central bank generates an open-market operation revenue. This revenue can turn negative if, for example following an increase in interest rate, the value of long-term assets purchased declines while the cost of short-term liabilities increases.

The rise in interest rate has led to losses for major central banks in recent years. In 2023, these losses amounted to \$114.3 billions for the Fed (0.4% GDP), €12.4 billions for the Banque de France (0.4% GDP), £37 billions for the Bank of England (1.6% GDP), and comparable losses ranging from 0.5% to 1.5% GDP for other major central banks. These losses should be considered in light of the profits realized during the zero lower bound period, but they may surpass them, as illustrated by the Bank of England's projections.[4] To offset these losses, central banks rely on existing buffers, drawing on reserves stock (as long as available), claims against future profits, or direct recapitalization by the government of shareholders (see Bunea et al. (2016)). As illustrated in Figure 1 for the Fed and the Banque de France, central banks may transfer profits to the Treasury, but not losses.

Figure 1: **Net revenue and remittance to the Treasury for Fed and Banque de France**



The Federal Reserve Act specifies that "during a period when earnings are not sufficient to provide for those costs, a deferred asset is recorded. The deferred asset is the amount of net earnings the Reserve Banks will need to realize before their remittances to the U.S. Treasury resume".[5] Therefore, remittances to the Treasury are positive during the expansion, and null until the Fed repays its claims. With multiple shareholders, the ECB's status is less clear, but it stipulates that "in the event of a loss incurred by the ECB, the shortfall may be offset against the general reserve fund of the ECB and, if necessary, [...], against the monetary income of the relevant financial year in proportion and up to the amounts allocated to the national central banks".[6]

Therefore, the asymmetric transmission of central bank profits and losses alters the time path of fiscal revenue for the government and consequently the required fiscal adjustments. This suggests that the budget constraints of the central bank and the Treasury cannot be consolidated, and should be kept separated, yielding different tra-

[4]https://www.bankofengland.co.uk/asset-purchase-facility/2023/2023-q3

[5]https://www.federalreserve.gov/newsevents/pressreleases/other20230113a.htm

[6]https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:12016E/PRO/04

jectories of taxes and public debt than with consolidated budgets. Other arguments for maintaining separated budget constraints include strategic behavior when distinct shareholders exist (see Bassetto and Sargent (2020)), the necessity for independence and transparency (see C. A. Sims (2004)), and the political incentive to report profits (see Goncharov, Ioannidou and Schmalz (2023)).

Then, in this paper, balance sheet policies transmit through two channels. First, it modifies the fiscal distortions in the new steady state, stimulating the economy through anticipatory effects. Second, it generates a transitory loss at the end of the ZLB period, which alters the trajectory of taxes and is non-neutral.

2 Complete model

Having discussed the two theoretical channels of balance sheet expansion, we turn to the ingredients of our complete model. First, we introduce a **government** with a debt, held either by households or the central bank, that levies taxes to repay the interests. Second, a **central bank**, usually running a conventional interest rate policy, but which is forced to shift to an asset purchase program in response to a shock pushing the economy to the Zero Lower Bound. Third, **households** to hold the money created by the central bank, which enters the utility function, with a satiation point to prevent money from strictly dominating other assets at the ZLB. Fourth, the **heterogeneous agent model** à la Aiyagari (1994) implies that the Ricardian equivalence and then the neutrality of the time profile of taxes do not hold, and enables a balance sheet expansion to generate a permanent decrease in the real interest rate at the final steady state. Finally, monetary policy is not neutral in our model: we use the New Keynesian framework with **firms** in monopolistic competition facing price adjustment costs.

2.1 Households

The economy is populated by an infinite amount of heterogeneous households with idiosyncratic productivities. They maximize intertemporal utility, choosing consumption C, labor N, asset A and money M, subject to their budget constraint, their idiosyncratic productivity process z, a preference shifter Z_t,[7] and a borrowing constraint. The

[7] We use it later to reach the ZLB: when Z_t increases, households are more patient, save more and consume less, which creates a negative demand shock and deflation, pushing the central bank to decrease the nominal rate.

program of household i is the following:

$$\max_{\{C_{i,t},N_{i,t},A_{i,t},M_{i,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t Z_t \left(\frac{C_{i,t}^{1-\sigma}-1}{1-\sigma} - \nu \frac{N_{i,t}^{1+\psi}}{1+\psi} + \chi \frac{\min\left\{\bar{m}, \frac{M_{i,t}}{P_t}\right\}^{1-\mu}}{1-\mu} \right)$$

such that

$$\begin{cases} P_t C_{i,t} + A_{i,t} + M_{i,t} = (1+i_{t-1})A_{i,t-1} + M_{t-1} + (1-\tau_t)W_t z_{i,t} N_{i,t} + \Pi_t(z_{i,t}) \\ A_{i,t} \geq 0 \\ z_{i,t} = e^{x_{i,t}} \ , \ x_{i,t} = \rho_z x_{i,t-1} + \epsilon_{i,t} \ , \ \epsilon_{i,t} \sim \mathcal{N}(0, \sigma_z^2) \end{cases}$$

The first constraint is the budget constraint in nominal terms, with P_t the price of the consumption good, i_t the nominal return on public debt, τ_t the labor tax, W_t is the nominal wage and $\Pi_t(z_{i,t})$ the nominal dividend from the firm.[8] The second equation is the borrowing constraint, and the third is the idiosyncratic productivity process of persistence ρ_z and variance σ_z^2.

Denoting λ and κ the multipliers associated with the budget and borrowing constraints and introducing the ratio $\eta = \frac{\lambda}{\kappa}$, we obtain the household money demand:

$$\frac{M_t}{P_t} = \min\left\{\bar{m}, C_t^{\frac{\sigma}{\mu}} \left(\chi \frac{1+i_t}{i_t+\eta_t}\right)^{\frac{1}{\mu}}\right\} \tag{2}$$

Money demand is an increasing function of the consumption (which would also be the case with a cash-in-advance (CIA) constraint, as in Auerbach and Obstfeld (2005)), and a decreasing function of the interest rate (which allows to have a varying velocity of money, as opposed to the CIA modelling). Moreover, the min operator introduces a satiation point, a feature important to induce the zero lower bound. Without satiation, the money demand would become infinite when we approach the ZLB, and households would convert all their assets into money. In a one period bond model like ours, the financial market would then not clear. Another interesting specification of this money demand is the presence of the borrowing constraint: when $\eta_t > 0$ (*i.e.* when the borrowing constraint on A is binding), the household reduces its money holding to self-insure. The implication of this term is that even at the ZLB, all agents do not stand at the satiation point, because the marginal utility of money is finite even before reaching satiation, as you want to reduce your money holding to smooth your consumption (which is consistent with Abo-Zaid and Garin (2016), who show that satiating the economy with real money requires a negative nominal interest rate).

[8] We assume the real profit of the firm Π_t is distributed proportionally to productivity, so that $\Pi_t(z_{i,t}) = \Pi_t \frac{z_{i,t}}{\int_i z_{i,t} \mathrm{d}i}$.

2.2 Firms

Intermediate goods producers in monopolistic competition maximize their intertemporal profit, choosing intermediate output y, price p and labor n, subject to the production function, the Rotemberg price adjustment cost and the demand schedule from the competitive representative final goods producer.[9] The program of the firm j is the following:

$$\max_{\{y_{j,t},n_{j,t},p_{j,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} Q_{0,t}\left[p_{j,t}y_{j,t} - W_t n_{j,t} - P_t\Theta_t\right]$$

such that

$$\begin{cases} y_{j,t} = n_{j,t} & \text{(Production function)} \\ \Theta_t = \frac{\theta}{2}\left(\frac{p_{j,t}}{p_{j,t-1}} - 1\right)^2 Y_t & \text{(Rotemberg cost)} \\ y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\epsilon} Y_t & \text{(Demand)} \end{cases}$$

with $Q_{0,t}$ the discount factor of the firm,[10] θ the price rigidity factor, ϵ the elasticity of substitution across goods and Y_t the aggregate output. Assuming a symmetric equilibrium in which all firms set the same price and choose the same amount of labor, and denoting the real wage $w_t = \frac{W_t}{P_t}$ and the gross inflation rate $\pi_t = \frac{P_t}{P_{t-1}}$, we obtain the New-Keynesian Phillips curve:

$$\frac{\epsilon}{\theta}\left(w_t - \frac{\epsilon-1}{\epsilon}\right) + \mathbb{E}_0\left[\frac{1}{r_{t+1}}\frac{Y_{t+1}}{Y_t}\pi_{t+1}(\pi_{t+1}-1)\right] = \pi_t(\pi_t - 1)$$

that relates the current price inflation rate to the expected price inflation rate and the current real marginal cost.

2.3 Government

The government has two sources of expenditures: the repayment of the real debt d_{t-1} and the constant public expenditures $\bar{G}$. To finance these expenditures, it receives the profit s^{CB} of the central bank and adjusts the labor income tax rate τ and the public debt d_t to clear the following budget constraint in real terms :

$$(1+r_t)d_{t-1} + \bar{G} = d_t + s_t^{CB} + \tau_t w_t \int_i z_{i,t} n_{i,t} \mathrm{d}i \tag{3}$$

[9]For the sake of brevity, we do not develop the firm side here: for a complete derivation of the intermediate inputs structure and the New-Keynesian Phillips curve, see Gali (2008).

[10]We assume that the discount factor of the firm is $Q_{0,t} = \prod_{k=0}^{t} \frac{1}{1+r_k}$.

We follow Airaudo (2023) and assume the tax rate follows a rule given in Equation 4, to stabilize the public debt at its target $\bar{d}$, with an autoregressive coefficient ρ_τ and an elasticity of tax deviations to debt deviations characterized by the parameter γ_d:

$$\tau_t - \bar{\tau} = \rho_\tau(\tau_{t-1} - \bar{\tau}) + (1 - \rho_\tau)\gamma_d(d_{t-1} - \bar{d}) \tag{4}$$

During the transition, following a negative shock, the tax rate and the public debt will increase. On the long run, if the interest rate is lower or if the central bank profit is higher, which will be the case in our experiment, the labor tax rate decreases, playing a significant role in shaping the effect of the balance sheet expansion.

2.4 Central Bank

2.4.1 Rules for interest rate, balance sheet and money supply

The central bank has two instruments: the nominal interest rate in "normal" times, and the size of the balance sheet when the zero lower bound is binding, associated with change in money supply. The central bank controls the nominal interest rate according to a Taylor rule subject to the zero lower bound:

$$i_t = \max\{0, \bar{i} + \varphi(\pi_t - \bar{\pi})\}$$

with $\varphi > 1$. Moreover, the central bank can change the size of its balance sheet in a discretionary manner:

$$d_t^{CB} = d_{t-1}^{CB} + \Delta d_t^{CB}$$

During episodes of ZLB, this change in balance sheet size is associated with changes in money supply:

$$m_t = \begin{cases} \text{Identified by money demand at } i_t & \text{if } i_t > 0 \\ m_{t-1} + \Delta d_t^{CB} & \text{if } i_t = 0 \end{cases}$$

Outside the ZLB, since the nominal anchor is given by the Taylor rule on the nominal interest rate, the money supply is identified by money demand in order to support the nominal interest rate implied by the policy rule. However, at the ZLB, we assume a rule for money supply: money supply is kept constant in real terms, except money created for Quantitative Easing purposes. Therefore, in our experiment, we assume that when the ZLB is binding, the central bank creates money to buy public debt; while in our counterfactual scenario, the central bank keeps constant the real value of money, and we assume $\Delta d_t^{CB} = 0$.

2.4.2 Profits and losses of the Central Bank

In our model, the central bank has two sources of revenue: the inflation tax revenue, and the asset revenue through its public debt holding. Denoting Ψ_t^{CB} the real profit or loss of the central bank, we have in nominal terms:

$$P_t \Psi_t^{CB} = \Delta M_t + (1 + i_{t-1}) D_{t-1}^{CB} - D_t^{CB}$$

In real term, the profit becomes

$$\Psi_t^{CB} = \underbrace{r_t d_t^{CB}}_{\text{Asset revenue}} \underbrace{+ m_{t-1}\left(1 - \frac{1}{\pi_t}\right)}_{\text{Inflation tax}} + \underbrace{\Delta m_t}_{\text{New money}} - \underbrace{\Delta d_t^{CB}}_{\text{Asset purchase}}$$

At the steady state with $\Delta d_t^{CB} = \Delta m_t = 0$, the profit of the central bank is positive; during the transition, it may become negative if money level drops, for example if the central bank abandons the control of money supply at the end of the ZLB period.

One important feature of our model is the possibility to have $\Psi_t^{CB} < 0$, which means losses for the central bank. To deal with these losses, we consider two scenarios: the "**CB securities**" scenario, which is our benchmark, and the "**Treasury support**" scenario. The "**CB securities**" scenario assumes an asymmetric fiscal-monetary interaction, where gains are transmitted to the Treasury, but not losses. In this scenario, if the profit is positive, it is transferred to the Treasury through s_t^{CB}; however, if the profit is negative, the central bank emits CB securities, denoted X_t^{CB}, to cover the loss, and stop transferring profit to the Treasury until this particular debt is fully repaid. This scenario can be seen as the "deferred asset" system used by the Fed. The transfer to the Treasury s_t^{CB} and the CB securities value X_t^{CB} are given by the following system:

$$\begin{cases} s_t^{CB} = \max\left\{0, \Psi_t^{CB} - (1 + r_t) X_{t-1}^{CB}\right\} \\ X_t^{CB} = (1 + r_t) X_{t-1}^{CB} + s_t^{CB} - \Psi_t^{CB} \end{cases}$$

In this scenario, our story is the following. When the ZLB period is over, the central bank switches back to an interest rate rule, and abandons the control of the money supply. The money level suddenly drops, as money is now dominated in return, creating a loss for the central bank. The central bank emits CB securities, and retain future earnings to repay its debt. When the debt is fully repaid, the transfer to the Treasury becomes positive again.

Our alternative scenario is the "**Treasury support**" scenario. In this case, we assume a symmetric fiscal-monetary interaction: both the gains and the losses are transferred to the Treasury, so that s_t^{CB} can be negative, and the central bank never issues CB securities:

$$\begin{cases} s_t^{CB} = \Psi_t^{CB} \\ X_t^{CB} = 0 \end{cases}$$

In this scenario, at the end of the ZLB period, the central bank makes losses, which are transmitted to the Treasury. According to its fiscal rule, the governement increases the labor tax rate and the public debt, so that the transmission of the loss towards households is more direct.

2.5 Market clearing conditions and equilibrium

Finally, to close the model, we have the following market clearing conditions:

$$\begin{cases} n_t = \int_i z_{i,t} n_{i,t} \mathrm{d}i & \text{(Labor)} \\ Y_t = \bar{G} + \Theta_t + \int_i c_{i,t} \mathrm{d}i & \text{(Goods)} \\ d_t + X_t^{CB} = d_t^{CB} + \int_i a_{i,t} \mathrm{d}i & \text{(Public debt)} \\ m_t = \int_i m_{i,t} \mathrm{d}i & \text{(Money)} \end{cases}$$

An equilibrium in this economy is defined as paths for households decisions $\{C_t, N_t, A_t, M_t\}_{t\geq 0}$, firms decisions $\{y_t, p_t, n_t, \Pi_t\}_{t\geq 0}$, prices $\{r_t, w_t, \pi_t\}_{t\geq 0}$, fiscal and monetary policies $\{\tau_t, i_t, d_t^{CB}\}_{t\geq 0}$, and aggregate quantities, such that, at every t, (i) households and firms maximize their objective functions taking as given equilibrium prices and taxes, (ii) the government and central bank budget constraints hold, and (iii) all markets clear.

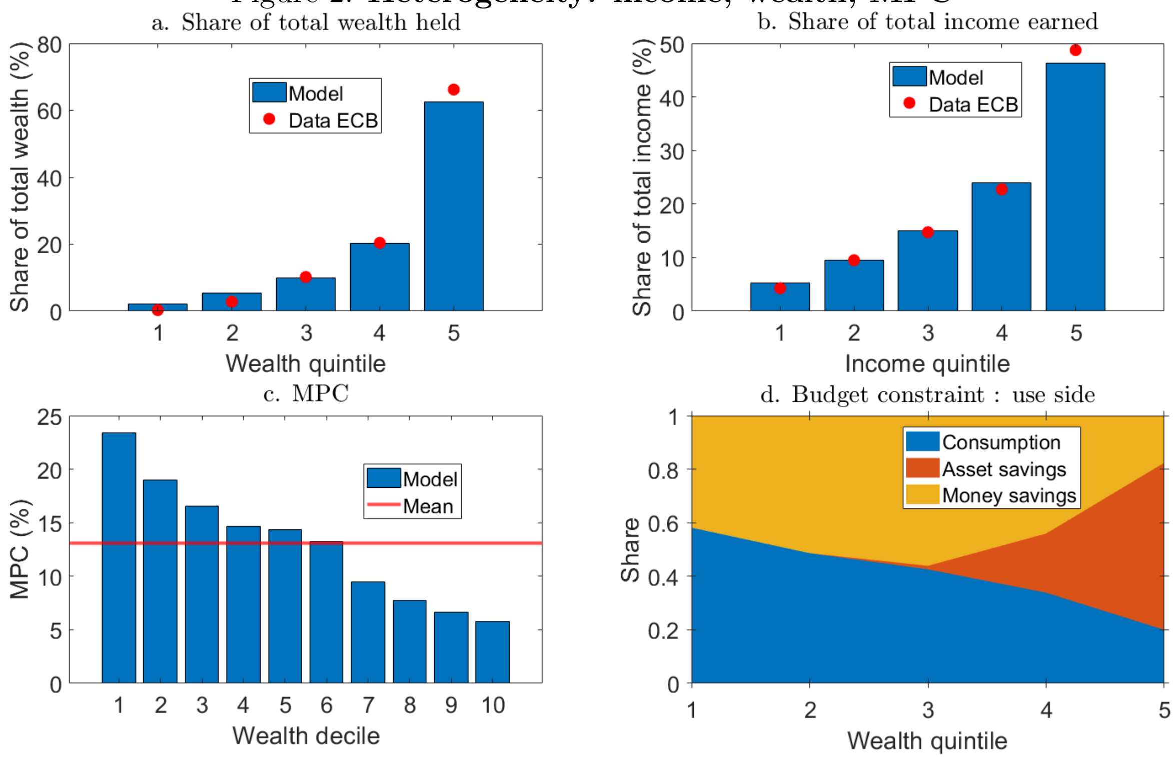
3 Calibration

We calibrate the model using Euro Area (EA) aggregate and micro data. We detail the calibration for households, especially related to the heterogeneity and the money demand. The firm, governement and central bank parameters are standard. All parameters are gathered in Table 1.

Money demand: the money utility scaling χ is set to have $\frac{m}{c} = \frac{\int_i m_i \mathrm{d}i}{\int_i c_i \mathrm{d}i} = 1.05$, which was the ratio between monetary aggregate M1 and annual consumption in Euro area before the implementation of QE in 2015 (see Figure 10 in Appendix A.2, top panel). The satiation level $\bar{m}$ is set to 1.2: at this level, 39% of households are money-satiated at the original steady state. A lower (higher) satiation point implies more (fewer) agents at the satiation point, and a smaller (greater) shock brings about the ZLB. Finally, the curvature of the utility function μ is used to calibrate the semi-elasticity of money demand to the interest rate. We follow the Ireland (2009) methodology and estimate the semi-elasticity of money demand in Europe before 2013 and the first ZLB event (see A.2 for details) to be equal to 4.5%, which implies $\mu = 1$ in our model.

Heterogeneity: the persistence parameter ρ_z and the standard deviation σ_z of the idiosyncratic productivity shock are calibrated to match the share of total assets held across the total assets distribution, and the share of total income earned across the income distribution, in the Euro area (see sources, definition and data in Appendix A.2). As shown in panels a and b of Figure 2, we perform fairly well across both dimensions, with a very concentrated wealth distribution and a more equal income distribution. The accuracy is smaller at the extremities of the distribution: since money enters the utility function, even poor agents hold money and then have positive wealth. We show in panel c the resulting marginal propensities to consume across the wealth distribution in our model: it goes from 23% for the 10% poorest to 6% for the 10% wealthiest (13% on average). Finally, we show in panel d the use-side (*i.e.* the left part of the budget constraint) across the wealth distribution: the share of consumption decreases with wealth, the 60% poorest households hold all their saving in money, and the share of bonds in total saving increases only for the 40% wealthiest households.

Figure 2: **Heterogeneity: income, wealth, MPC**



Lecture: **a)** The wealthiest 20% in the Euro area hold 66% of the total wealth in the data, and 62% in our model. **b)** The 20% lowest income earn 4.3% of the total income in the data, and 5.3% in our model. **d)** The share of consumption in the "use-side" of the budget constraint decreases with wealth, while the share of bonds in total saving increases.

Other households parameters: as in Kaplan, Moll and Violante (2018), we choose the inverse intertemporal elasticity of substitution σ to be equal to 1 and a unitary inverse Frisch elasticity ψ. The discount factor β is set to target a 3.5% nominal annual interest rate, the average ECB refinancing rate between 2000 and 2008. The labor disutility scaling ν allows to normalize the initial output to 1.

Table 1: Parameter values and steady-state targets.

Parameter	Description	Value	Notes
Households			
β	Discount factor	0.945	annual nominal interest rate: 3.5%
σ	Curvature of utility w.r.t. C	1	intertemporal elasticity of substitution: 1
ν	Labor disutility scaling	1.3	steady state initial output: 1
ψ	inverse Frisch elasticity	1	Frisch elasticity: 1
χ	weight of money in the utility function	0.07	ratio annual consumption / M1 : 1.05
μ	Curvature of utility w.r.t. m	1	Initial semi-elasticity of m to i: 4%.
$\bar{m}$	real money satiation level	1.2	share of people at the satiation : 39%
ρ_z	persistence of productivity shock	0.92	see text
σ_z	variance of productivity shock	0.25	see text
Firm			
ϵ	elasticity of substitution	7	markup: 14%
θ	price adjustment cost parameter	150	average price duration: 3 quarters
Government and central bank			
$\bar{G}$	real government expenditures	0.28	income tax rate: 30%
γ_d	tax reaction to debt deviation	0.1	Airaudo (2023)
ρ_t	tax persistence	0.9	Airaudo (2023)
$\bar{d}$	real debt	1	average debt-to-output ratio in Europe: 100%
ϕ	reaction to inflation	1.5	standard value
$\bar{\pi}$	long-run inflation target	1.02	net inflation rate: 2%

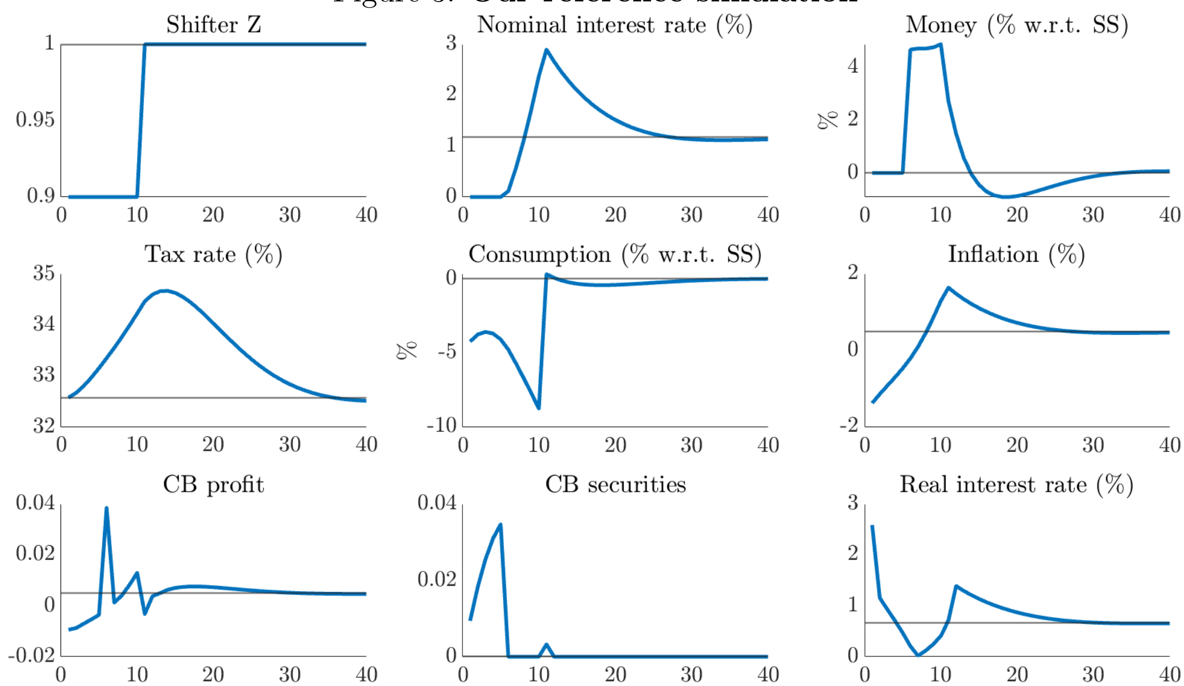
4 Numerical results

In this section, we simulate the effect of balance sheet policies following a negative demand shock. In subsection 4.1, we consider a negative demand shock that pushes the economy at the ZLB, in the absence of balance sheet policy. In subsection 4.2, we introduce a balance sheet expansion (quantitative easing, QE) during the ZLB, followed by different scenarios of balance sheet reduction (quantitative tightening, QT). In subsection 4.3, we study the aggregate and distributive welfare effects of each policy. Finally, in subsection 4.4, we consider an alternative fiscal-monetary interaction to deal with CB losses.

4.1 Negative demand shock and ZLB

The central bank engages in a balance sheet expansion because it can no longer stimulate the economy by lowering the nominal interest rate, which is constrained by the zero lower bound. To simulate this scenario, we introduce an unexpected, deflationary negative shock in our economy, that pushes the interest rate to the zero lower bound through the Taylor rule. We follow Gali (2020) and introduce a negative demand shock through preferences, with preference shifter Z_t that multiplies the utility. When Z_t decreases over a certain period, it implies that current consumption provides less utility, leading households to consume less and save more. This negative demand shock triggers a recession and results in a binding zero lower bound on the nominal interest rate. We consider a 10-quarter shock for Z, as shown in Figure 3, which drives the nominal interest rate to zero. In our simulations, the ZLB period lasts 7 quarters. In our reference scenario, the size of the balance sheet stays constant, and since the Taylor rule is temporarily replaced by a money rule, this also means that money supply stays constant in real terms. When the ZLB period is over, the central bank abandons the direct control of money supply, so the money level increases by 4%, as the nominal interest rate is temporarily lower than its steady state value.

Figure 3: **Our reference simulation**



Lecture: Money and consumption are expressed in percentage deviation from their steady-state values. All other variables are expressed in real value.

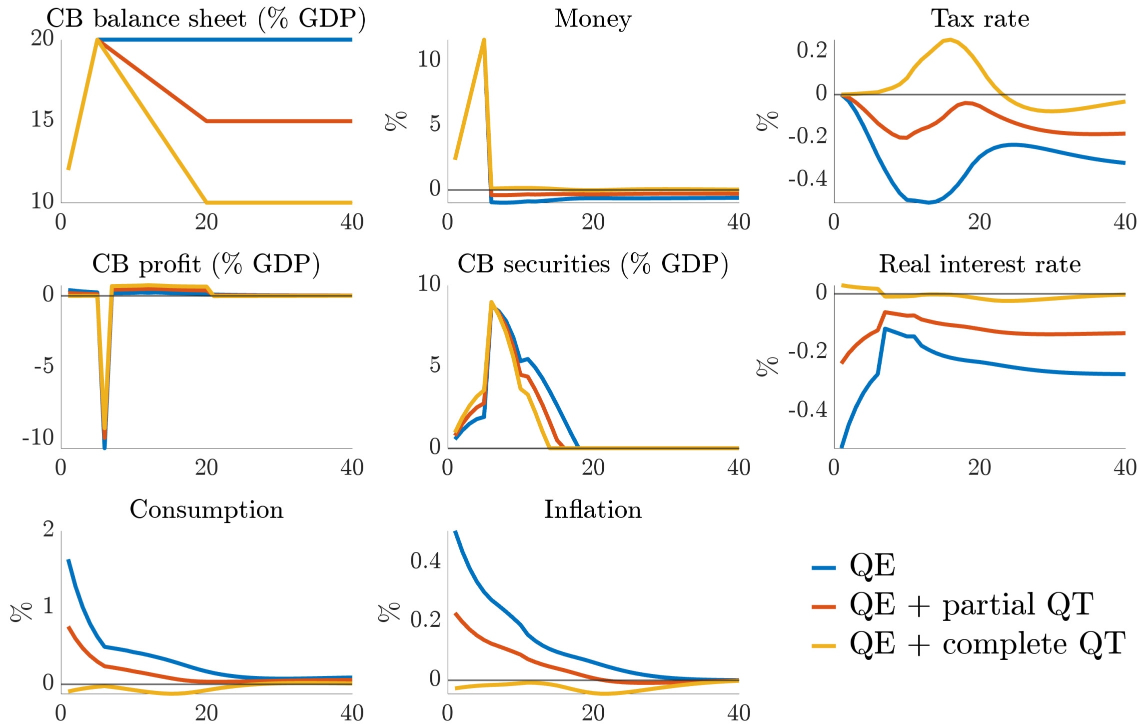
The reduction of consumption, output and labor supply forces the government to raise the tax rate. For the central bank, the deflation leads to negative seigniorage revenue, which must be offset by the issuance of central bank securities. This debt is fully repaid at the end of the ZLB period, with the jump in money supply and then in seigniorage.

This negative demand shock with no balance sheet policy constitutes our reference scenario. We then allow the central bank to implement a Quantitative Easing program during the ZLB, and compare the outcome with this reference scenario.

4.2 Balance sheet policies' effects

In this section, we add balance sheet policies to the negative demand shock described above. When the ZLB is binding, the central bank controls the money supply, and creates money to purchase public debt. In our experiment, we consider that the central bank balance sheet goes from 10 to 20% of GDP and total public debt stays constant at 100% of GDP. At the exit of the ZLB, we explore different scenarios for the balance sheet normalization. The blue curve in Figure 4 is the "**QE without QT/Permanent QE**" scenario: it is the polar case where the central bank keeps constant in real terms its balance sheet forever. The opposite polar case is the "**QE with complete QT**" scenario in yellow: the central bank gradually sells all its assets through a quantitative tightening program, and goes back to a 10% balance sheet. Finally, the red curve is the middle case, the "**QE with partial QT**" scenario: the central bank sells half of the assets purchased during QE and keeps the other half forever, converging towards a new balance sheet equal to 15% of GDP.

Figure 4: **IRF with different QT scenarios (vs reference)**



Lecture: Money and consumption are expressed in percentage deviation from reference scenario values. Tax rate, real interest rate and inflation rate are expressed in % base points deviation from reference scenario. CB profit is expressed in value change from reference scenario. CB balance sheet, CB securities are expressed in real value.

The Figure 4 represents the impulse response function associated with each of these balance sheet policies, compared to the reference scenario with no asset purchase program. In all scenarios, the balance sheet expansion, and hence the money supply expansion, are of similar size. At the end of the ZLB, when the central bank abandons the control on money and reverts to a Taylor rule, households dissave their excess money, which starts to yield less than public debt. This triggers a substantial seigniorage loss for the central bank. Losses are covered by temporary CB securities emissions and repaid over time by future seigniorage and asset revenues, and asset sale depending on the normalization scenario.

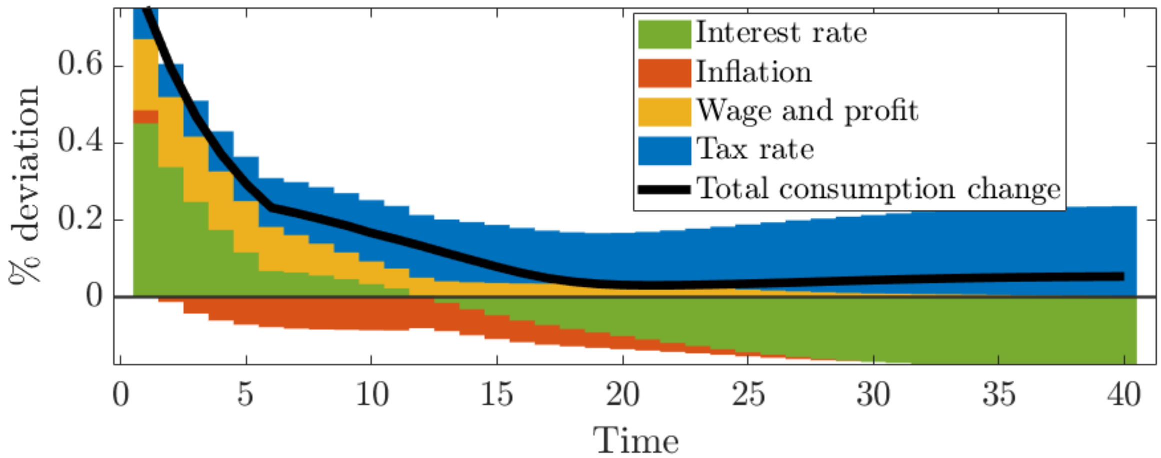
What crucially differs in these three scenarios is the QT program implemented at the end of the ZLB, and the anticipation of the new steady state reached. The scenario where **QE is not reversed**, allows to reach a steady state with a smaller interest rate (-0.25%), a smaller debt burden, and then a smaller tax rate (-0.3%). Households anticipate this new steady state and increase their consumption, even during the ZLB where QE should be neutral. Consumption increases by 1.5% on impact, triggering a 0.5% increase in inflation. This result is closely related to Auerbach and Obstfeld

(2005), who show that trading money for interest bearing public debt reduces future debt-service requirement, and hence distorsions of the requisite taxes, which creates by anticipation a positive effect on consumption and inflation even at the ZLB. Here, we extend their framework, by replacing their money rule by a switch between a Taylor rule on the interest rate and a money supply rule at the ZLB.

On the opposite, if **QE is fully reversed by QT**, the positive anticipation effect disappears, and consumption and inflation slightly decline on impact. The effect is small, but not null: the fact that a temporary balance sheet expansion is not fully neutral is due to the non-Ricardian structure of our economy, as constrained households cannot smooth future increases in tax rate induced by the lack of transfers from the central bank to repay its securities. Therefore, we provide an additional non-neutrality channel to the work of Benigno and Nistico (2020), who derive analytical conditions to break Wallace (1981) irrelevance result in a representative agent model. Indeed, in our non-Ricardian economy, the change in time path for public debt and taxes is not neutral, as financial markets are imperfect and households cannot smooth their consumption perfectly.

A more realistic scenario lies between these two polar cases. The partial balance sheet reduction is relevant because, as shown by Ferguson, Schaab and Schularick (2015), "central banks rarely reduce the size of their balance sheets in nominal terms after large expansion episodes. Reductions are predominantly achieved in real terms by holding nominal positions stable for some time". As the size of central bank balance sheets is never equal to zero and takes decades to decrease (as shown in Appendix A.2), we may expect households to anticipate that balance sheet policies will not be fully reverted shortly. Two forces are fighting: the convergence towards a permanent new steady state with smaller tax and interest rates, and the temporary cost of central bank losses. In an intermediate scenario, with **QE partially reversed by QT**, the consumption response is still positive, as the tax rate and the real interest rate stay below their initial level, even if they increase during the CB securities repayment period. To further understand this change in consumption, we need to quantify the contribution of each channel identified in equation 1. We then decompose the consumption change in this intermediate scenario into 4 parts: the change due to the changes in interest rate, inflation, labor tax rate and wages and profit (this last channel was not present in our introducing example without monopolistic competition). The decomposition is shown in Figure 5 :

Figure 5: **Decomposition of consumption change**



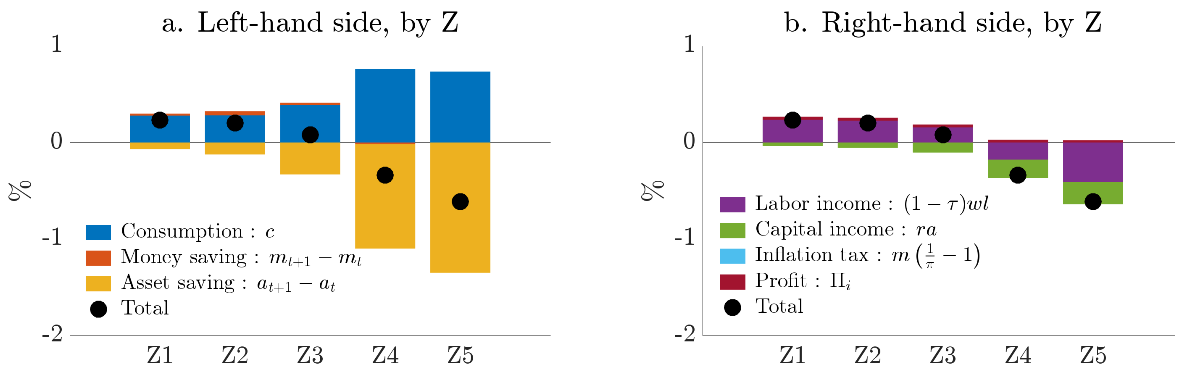
Lecture: Breakdown of consumption change in scenario **QE partially reversed by QT**. We compute numerically the partial derivative of individual consumption with respect to each aggregate variable: i_{t-1}, τ_t, w_t, Π_t.

The change in consumption is primarily explained by two factors: the change in interest rate, and the change in labor tax rate. On the one hand, on impact, the balance sheet policy reduces the interest rate, increasing short-run consumption through the Euler equation. On the long-run, however, the permanent decrease in interest rate lowers asset revenue and then households consumption. On the other hand, the permanent decrease in the distortive labor tax rate induces households to work more and then increases consumption. Quantitatively, the positive tax rate channel dominates the negative interest rate channel, so that consumption is higher at the new steady state. Another channel is the change in wages and profits, due to the New Keynesian monopolistic competition structure. Wages increase with inflation and profits slightly decrease: the overall change is still positive and contributes to an increase in aggregate consumption. Finally, the increase in inflation represents an increase in the distortive inflation tax for households, lowering the consumption. The sum of these channels is positive: consumption increases by 0.8% on impact, and is 0.05% higher on the long-run.

4.3 Welfare and distributive effects

We now turn to the analysis of welfare properties and distributive effects of our benchmark balance sheet policy, **QE partially reversed by QT**. The two main channels for this analysis are the decrease in interest rate and the decrease in labor tax rate following the balance sheet expansion. This creates an efficiency gain by reducing the labor distortion, and favors labor income at the expense of capital income. Moreover, the reduction of the quantity of assets available to households to self-insure pushes more households to the budget constraint, reducing their welfare. We first show in Figure 6 the static change in each component of the household budget constraint, between the steady states before and after the balance sheet expansion, ranking households by productivity quintiles.

Figure 6: **Decomposition of budget constraint changes, by productivity**

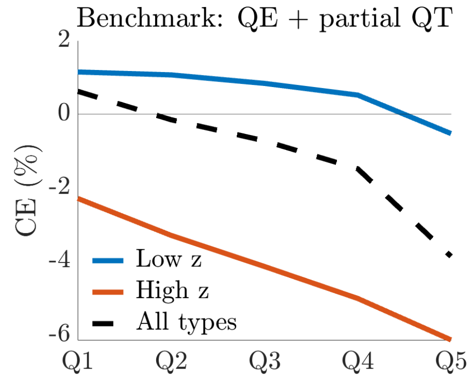


Lecture: we write the budget constraint as $c + (a' - a) + (m' - m) = (1 - \tau)wl + ra + m(1/\pi - 1) + \Pi$ and compute the change between the two steady states, ranking households by productivity quintiles.

Each household consumes more after the balance sheet expansion, but the total change in revenues and expenditures strongly depends on productivity. For low-productive, poor households, the increase in net wage dominates, while the decrease in interest rate does not have important effects as they do not hold many assets. The overall effect on revenue is positive. However, for high-productive high-income households, labor supply decreases because the substitution effect dominates. Therefore, the decrease in capital income combines with the decline in labor income, leading to a reduction in total income.

We now turn to the dynamic welfare analysis, by computing the welfare change along the transition. We follow Bhandari et al. (2021) and decompose the welfare gains into three components: aggregate efficiency, redistribution, and insurance. Efficiency captures the change in welfare due to changes in aggregate resources. Redistribution captures the welfare gains resulting from redistribution across agents. Insurance captures the change in household risk due to the change in self-insurance capacity. In our benchmark exercise, welfare decreases along the transition, with 73% of this decline resulting from the insurance component, 26% from the efficiency component and 1% from the redistribution component. However, this loss is unevenly distributed across households. We compute for each household the change in consumption equivalent (CE), *i.e.* the change in steady-state consumption that would make the household indifferent between the transition with and without the balance sheet policies.[11] In Figure 7, we show the change in CE by income quintile and by productivity, for low and high productivity levels.

Figure 7: **Welfare change by productivity**



Lecture: CE by disposable income quintile. Blue/red lines are for low/high productivity, and black line is the average.

As shown in Figure 7 for our benchmark experiment (QE partially reversed by QT), welfare improves for low-productive households and low-income households, while it decreases for richest households. Higher productivity types are disproportionately hit with respect to lower types. This is because our scenario induces a drop in permanent public debt emissions, leading to less aggregate savings. This exacerbates the income risk for high types, leading to a stronger drop in welfare. Moreover, the disposable income gradient is decreasing. This illustrates the income composition change in our economy: as the interest rate and the public debt available to households decrease, capital income decreases, which affects mostly rich households who are public debt holders. Conversely, the drop in the labor tax rate benefits disproportionately more poor households that rely on labor income. All in all, the **balance sheet expansion and its partial reversion induces a change from capital to labor income**, hitting more higher productivity types and wealthier households.

4.4 Addressing Central Bank losses: the fiscal-monetary mix

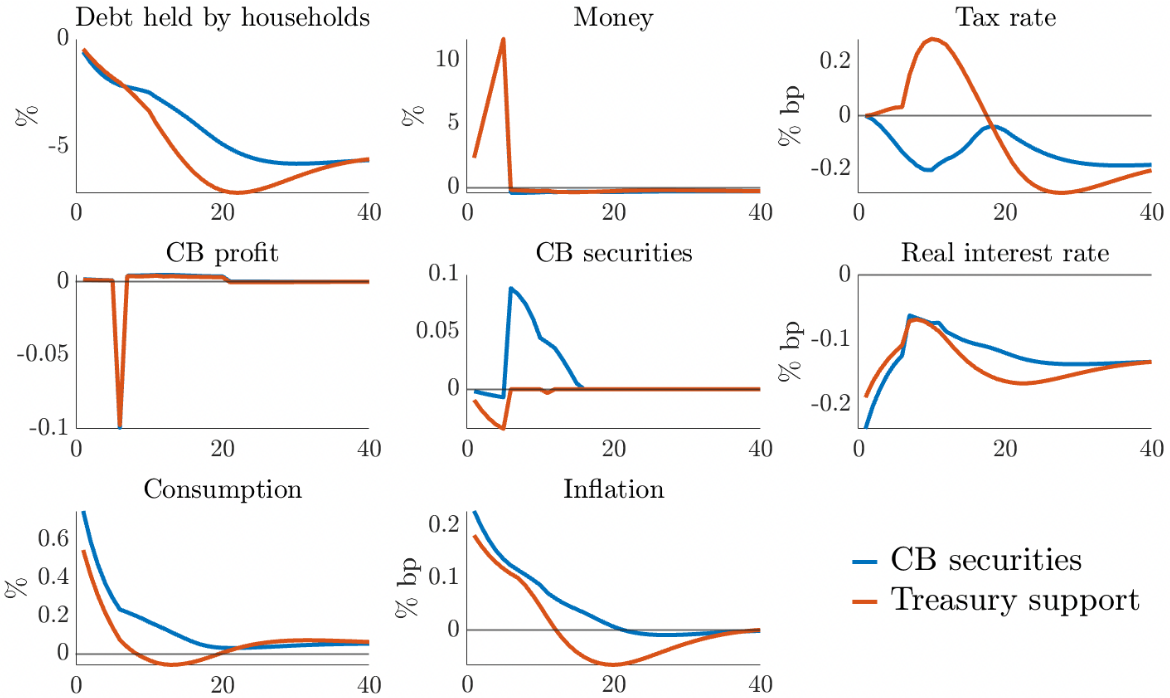
So far, we have focused on our benchmark scenario, "**CB securities' emissions**", where the central bank emits CB securities to cover its losses. We now compare an

[11] We compute for each initial wealth a_0 and productivity z_0 the following equality:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t^{\text{No QE}}(1+\text{CE}(a_0,z_0)), m_t^{\text{No QE}}, n_t^{\text{No QE}}\right)|a_0,z_0\right] = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t^{\text{QE}}, m_t^{\text{QE}}, n_t^{\text{QE}}\right)|a_0,z_0\right]$$

alternative way of smoothing losses with the same balance sheet policy: the "**Treasury support**" scenario, where losses are fully transmitted to the government. Figure 8 computes our benchmark balance sheet policy, with QE partially reversed by QT, and we show the results in deviation from the reference scenario. In Appendix A.3, we do the same comparison with our two polar balance sheet policies (QT+complete QT, and permanent QE).

Figure 8: **CB securities vs. Treasury support**



Lecture: Debt held by households, money and consumption are expressed in percentage deviation from reference values. Tax rate, real interest rate and inflation rate are expressed in base points differences from the reference scenario. CB profit and CB securities are expressed in change (GDP points) from reference scenario.

This figure illustrates that smoothing CB losses through future seigniorage revenue does not yield the same distorsions as future labor income taxation. In the "**Treasury support**" scenario, losses induce an increase in the tax rate through the fiscal rule defined in Equation 4. This leads to a smaller increase in consumption, labor supply and inflation, accentuating in turn the initial increase in the tax rate. At the same time, since there are fewer liquidity emissions than in the "**CB securities**" case, the real interest rate drops more, mitigating the drop in fiscal revenue. As inflation and the real interest rate decrease more in the medium term, consumption bounces back leading to a stronger decrease in the tax rate at the end of the transition. Therefore, **the fiscal and monetary interaction to deal with central bank losses has implications for the transmission of balance sheet policies**. In our framework, the transmission of

CB losses to the Treasury reduces the positive effect of balance sheet expansion, as it increases distortive taxes.

5 Conclusion

In most macroeconomic models, balance sheet policies are deemed neutral, both at the steady state or during a ZLB episode. Our paper challenges this feature, focusing on monetary-fiscal interactions. At the steady state, balance sheet expansion is not neutral as it gives more revenue to the central bank and then to the government, allowing it to modify pre-existing fiscal distortions. Moreover, it reduces assets available for households, reducing the interest rate in our incomplete market model where agents face a borrowing constraint. This non-neutrality propagates by anticipation along the transition. At the zero lower bound, agents anticipate that the world at the exit of the liquidity trap will be different, as the balance sheet expansion is expected to last or at least to not be fully reversed. They modify their consumption behavior, which stimulates the economy. Furthermore, we study a new transmission channel of balance sheet policies: the smoothing of central bank losses. In other words, the fiscal and monetary interaction to cover central bank losses affects economics aggregates. We find that emitting CB securities creates fewer distortions than transmitting losses to the Treasury. **The overall effect of balance sheet expansion depends on the anticipation of the future normalization, as well as the transmission of central bank losses**.

Our paper opens several research avenues. First, one important channel of quantitative easing (QE) is the increase in investment resulting from the decrease in interest rate. Increased government spending (or public debt) "crowds out" investment because it raises demand for savings, driving up interest rates and thereby reducing private investments. Therefore, introducing capital and equity into the model is a natural step forward, and will amplify the effect of balance sheet policies. Second, it appears that governments have relied on central bank support to expand public debt. A model with an endogenous public debt target could provide deeper insights into the fiscal-monetary policy mix. Third, our analysis focused on a closed economy. Extending our framework to an open-economy setting could reveal interesting trade-offs. With trade balance adjustments frictions, the macroeconomic effects of repurchasing public debt bonds held by domestic versus foreign investors may differ. These additional explorations would advance our understanding of the effects of balance sheet policies, which can offer valuable insights for policymakers and researchers in the future balance sheet reduction or zero lower bound periods.

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A Appendix

A.1 Proof

Step 1: we combine the budget constraint of the Treasury ($r\bar{d} = \tau wN + s^{CB}$), the budget constraint of the central bank ($s^{CB} = m\pi + rd^{CB}$) and the market clearing condition ($\bar{d} = d^H + d^{CB}$) to obtain the consolidated budget constraint :

$$rd^H = \tau wn(\tau, C) + \pi m\left(C, r(d^{CB})\right)$$

Derivating with respect to d^{CB}:

$$\frac{\mathrm{d}rd^H}{\mathrm{d}d^{CB}} = \frac{\mathrm{d}\tau n(\tau, C)}{\mathrm{d}d^{CB}} + \pi\frac{\mathrm{d}m\left(C, r(d^{CB})\right)}{\mathrm{d}d^{CB}}$$

Denoting $y_x = \frac{\partial y}{\partial x}$:

$$\frac{\mathrm{d}rd^H}{\mathrm{d}d^{CB}} = \frac{\mathrm{d}\tau}{\mathrm{d}d^{CB}}n + \tau\left(n_\tau\frac{\mathrm{d}\tau}{\mathrm{d}d^{CB}} + n_C\frac{\mathrm{d}C}{\mathrm{d}d^{CB}}\right) + \pi\left(m_r\frac{\mathrm{d}r}{\mathrm{d}d^{CB}} + m_C\frac{\mathrm{d}C}{\mathrm{d}d^{CB}}\right)$$

$$\iff \frac{\mathrm{d}rd^H}{\mathrm{d}d^{CB}} - \pi m_r\frac{\mathrm{d}r}{\mathrm{d}d^{CB}} = \frac{\mathrm{d}\tau}{\mathrm{d}d^{CB}}(n + \tau n_\tau) + (\tau n_C + \pi m_C)\frac{\mathrm{d}C}{\mathrm{d}d^{CB}}$$

Step 2: we have the aggregate budget constraint $C = Y = n(\tau, C)$. Derivating with respect to d^{CB}:

$$\frac{\mathrm{d}C}{\mathrm{d}d^{CB}} = \frac{\partial n}{\partial \tau}\frac{\mathrm{d}\tau}{\mathrm{d}d^{CB}} + \frac{\partial n}{\partial C}\frac{\mathrm{d}C}{\mathrm{d}d^{CB}}$$
$$\iff \frac{\mathrm{d}\tau}{\mathrm{d}d^{CB}} = \frac{\mathrm{d}C}{\mathrm{d}d^{CB}}\frac{1-n_C}{n_\tau}$$

Step 3: we combine both by replacing $\frac{\mathrm{d}\tau}{\mathrm{d}d^{CB}}$:

$$\frac{\mathrm{d}rd^H}{\mathrm{d}d^{CB}} - \pi m_r \frac{\mathrm{d}r}{\mathrm{d}d^{CB}} = \frac{\mathrm{d}C}{\mathrm{d}d^{CB}}\frac{1-n_C}{n_\tau}(n + \tau n_\tau) + (\tau n_C + \pi m_C)\frac{\mathrm{d}C}{\mathrm{d}d^{CB}}$$

$$\iff \frac{\mathrm{d}C}{\mathrm{d}(d^{CB})} = \frac{\frac{\mathrm{d}rd^H}{\mathrm{d}d^{CB}} - \pi m_r \frac{\mathrm{d}r}{\mathrm{d}d^{CB}}}{\frac{1-n_C}{n_\tau}(n+\tau n_\tau) + \tau n_C + \pi m_C}$$
$$= \frac{\frac{\mathrm{d}r}{\mathrm{d}d^{CB}}d^H - r + \frac{\mathrm{d}r}{\mathrm{d}d^{CB}}\left(-\frac{m_r}{m}\right)m\pi}{\frac{1-n_C}{\frac{n_\tau}{n}} + \tau + \pi m_C}$$

With $\epsilon_{y,x} = \frac{y_x}{y}$ the semi-elasticity of y with respect to x and multiplying by -1 to have positive numerator and denominator, we obtain the formula 1:

$$\iff \boxed{\frac{\mathrm{d}C}{\mathrm{d}(d^{CB})} = \frac{\overbrace{\left(-\frac{\mathrm{d}r}{\mathrm{d}(d^{CB})}\right)d^H}^{\text{Financial friction}} + \overbrace{\left(-\frac{\mathrm{d}r}{\mathrm{d}(d^{CB})}\right)|\epsilon_{m,r}|m\pi}^{\text{Inflation tax}} + r}{\underbrace{\frac{1-n_C}{|\epsilon_{n,\tau}|}}_{\text{Labor distortion}} - \tau - \pi m_C}}$$

The numerator is positive, as $\frac{\mathrm{d}r}{\mathrm{d}(d^{CB})} < 0$: if d^{CB} increases, less debt is available for households, so the interest rate must decrease. Therefore, we have $\frac{\mathrm{d}C}{\mathrm{d}(d^{CB})} > 0$ if the denominator is also positive. If we assume the utility function $u(c, n, m) = \ln c - \frac{n^{1+\psi}}{1+\psi} + \chi \ln m$, we obtain the labor supply $n = \left(\frac{(1-\tau)w}{c}\right)^{1/\psi}$ and the money demand $m = c\chi\frac{1+i}{i}$. This implies $n_c = -\frac{n}{c\psi} = -\frac{1}{\psi}$ (because $c = y = n$), $m_C = \frac{m}{c}$ and $\epsilon_{n,\tau} = -\frac{1}{\psi(1-\tau)}$. Then:

$$\frac{\mathrm{d}C}{\mathrm{d}(d^{CB})} > 0 \iff \frac{1-n_C}{|\epsilon_{n,\tau}|} - \tau - \pi m_C > 0 \iff \frac{1+\frac{1}{\psi}}{\frac{1}{\psi(1-\tau)}} - \tau - \pi\frac{m}{c} > 0$$
$$\iff (1-\tau)(\psi+1) - \tau - \pi\frac{m}{c} > 0 \iff \tau < \frac{1+\psi-\pi\frac{m}{c}}{2+\psi}$$

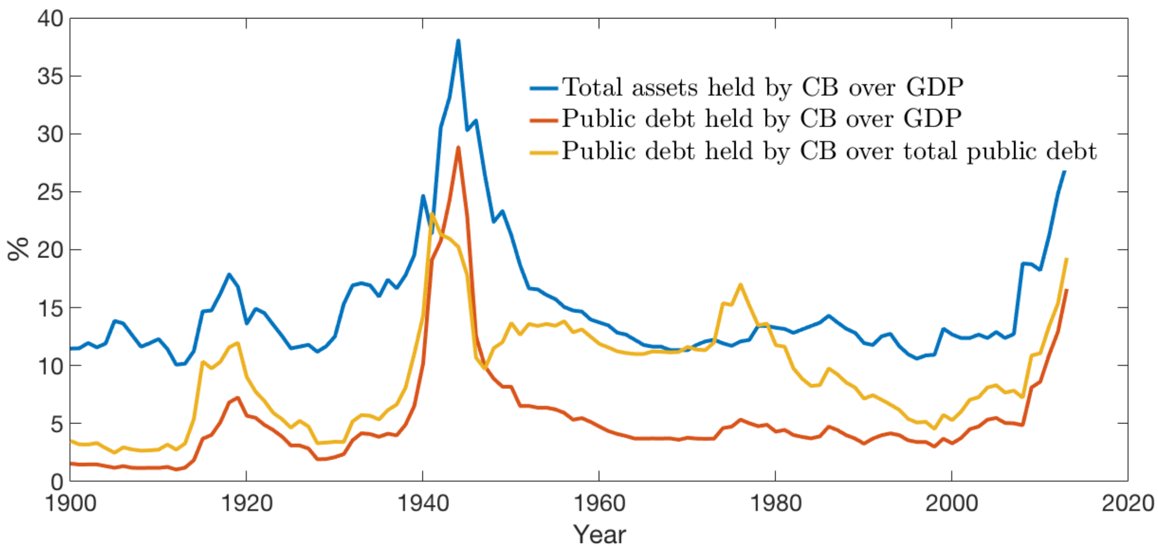
If we assume $\pi = 2\%$ and $\frac{m}{c} = 1$, $\frac{1+\psi-\pi\frac{m}{c}}{2+\psi}$ is superior or equal to 49%, so that for reasonible τ, this condition is respected, so that $\frac{\mathrm{d}C}{\mathrm{d}(d^{CB})} > 0$.

A.2 Data

Central bank balance sheet over time

Ferguson, Schaab and Schularick (2015) provide time series data on the average size of the central bank balance sheets,[12] between 1900 and 2013. We reproduce their graph in Figure 9, and show the average total assets and public debt held by central banks relative to the GDP, and the average public debt held by central banks relative to the total public debt.

Figure 9: **Average assets and government debt held by central banks**



Calibration of interest rate semi-elasticity of money demand

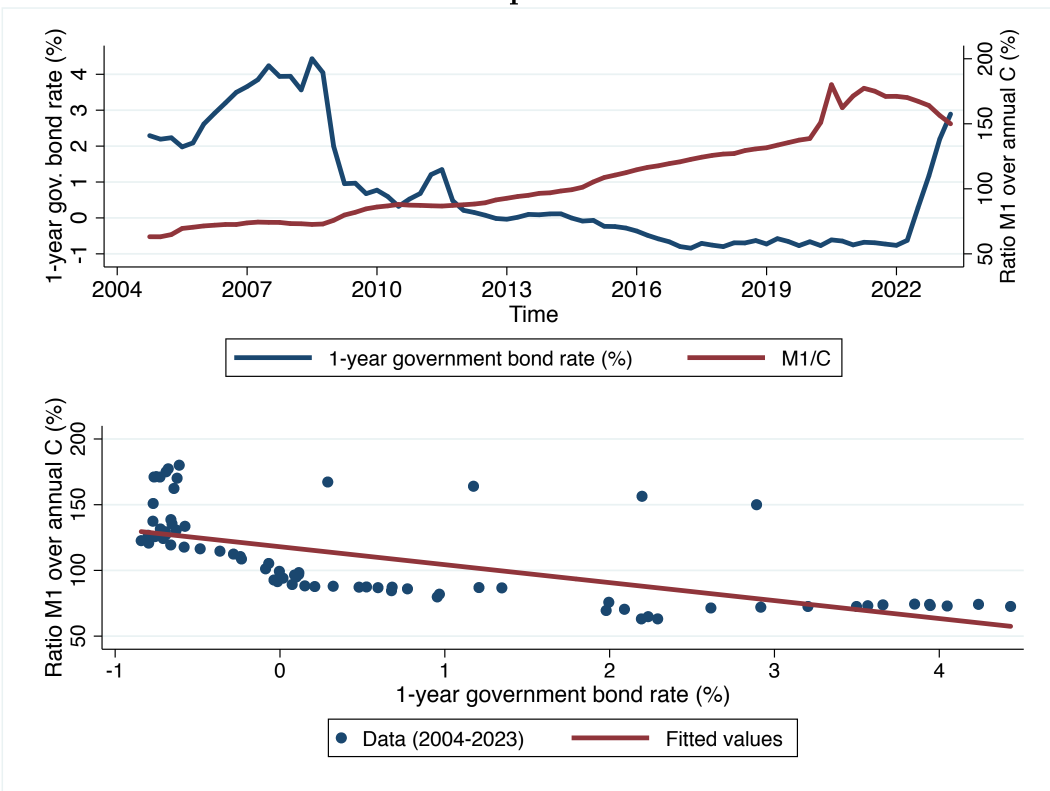
Ireland (2009) runs a regression in the United States between 1900 and 2006 to find the link between the nominal interest rate (measured by six-month Treasury bills rate) and the money demand (measured as the ratio between money and income), and find a low semi-elasticity around 2%, and even lower since 1980. We follow this approach in Figure 10 in Appendix A.2 (bottom panel) and relate the 1-year AAA Government bonds interest rate with the M1 over C ratio in the Euro area. If we consider the period between 2005 and 2023, we obtain a much higher result (11%) due to the decreasing trend of the interest rate and the increasing trend of M1/C. However, if we restrict our regression before 2013 and the first ZLB event, we obtain a semi-elasticity equal to 4.5%. Our semi-elasticity counterpart[13] must then lies between Ireland (2009) and the

[12] They use a dataset with 12 countries (Australia, Canada, Finland, France, Germany, Italy, Japan, Norway, Sweden, Switzerland, the United Kingdom and the United States) between 1900 and 2013. After 1999, they replace Finland, France, Germany and Italy by the ECB balance sheet.

[13] To compute the semi-elasticity in our model, we perform a partial equilibrium simulation: we

pre-2013 estimation in Europe: choosing $\mu = 1$ in our model gives us a semi-elasticity equal to 4% at the initial state, and 3.5% at the final steady state (because more agents are at the money satiation after the balance sheet expansion and therefore are less responsive to changes in the interest rate).

Figure 10: **1-year AAA Government bond interest rate and ratio M1 over annual consumption in Euro area**



Data for heterogeneity calibration

For Figure 2, we use the ECB Household Finance and Consumption Survey (wave 2017), including "country-representative data, which have been collected in a harmonised way in 22 European Union member states for a sample of 91,200 households". The definitions of variables are the following:

- *Wealth in data*: difference between total household assets (real assets including residence, vehicles, valuables objects, and financial assets including deposits, bonds, share) and total household liabilities (mortgages, loans, debt).
- *Wealth counterpart in the model*: sum of money holding and bonds holding.
- *Income in data*: sum of the employee income, self-employment income, public

change by a small amount the nominal interest rate, compute the value function iteration and the measure, and obtain the change in the money demand.

and private pensions, social transfers rental income from real estate property, and income from financial investments.

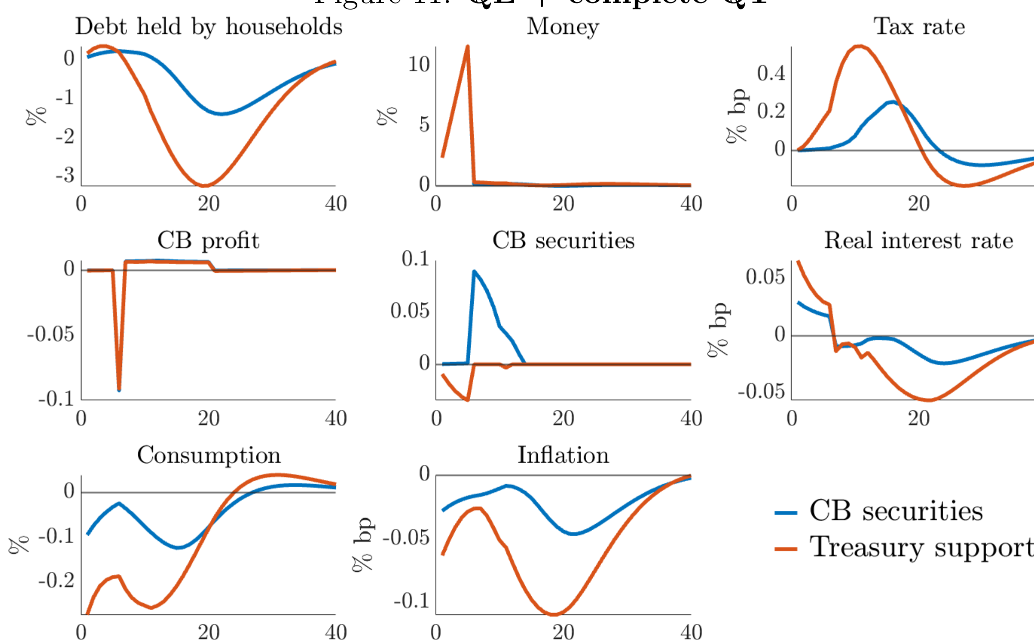
- *Income counterpart in the model*: sum of net labor income, capital income from bonds, return from money (negative inflation tax), transfers and profits.

A.3 Additional results

A.3.1 Fiscal and monetary interaction

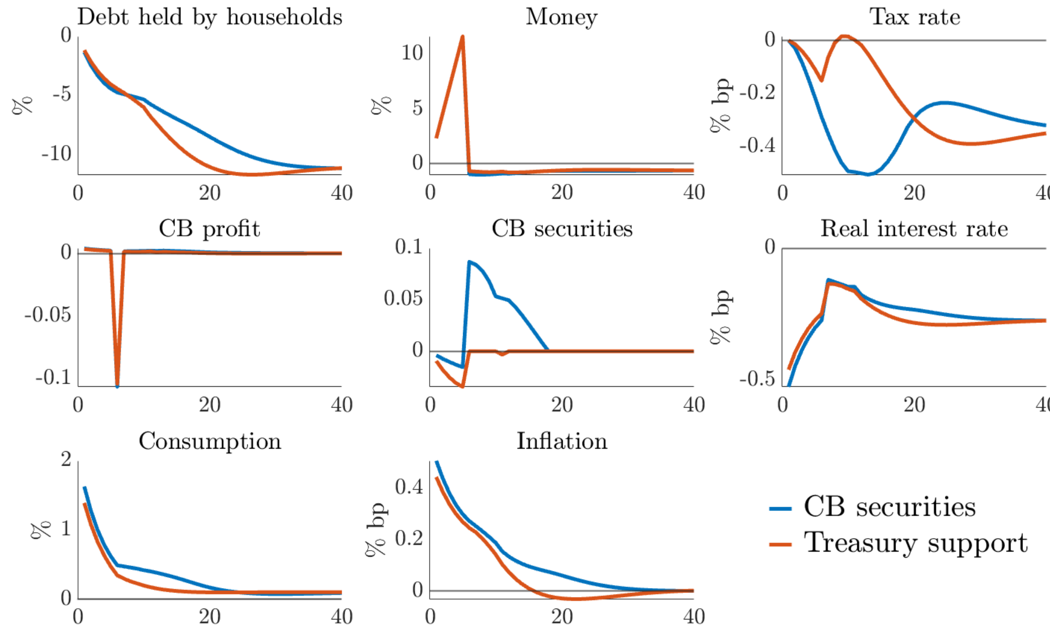
We show below the response of our two polar scenarios (QE and complete QT in Figure 11 and QE without QT in Figure 12) with our two ways to address central bank losses: CB securities, and Treasury support.

Figure 11: **QE + complete QT**



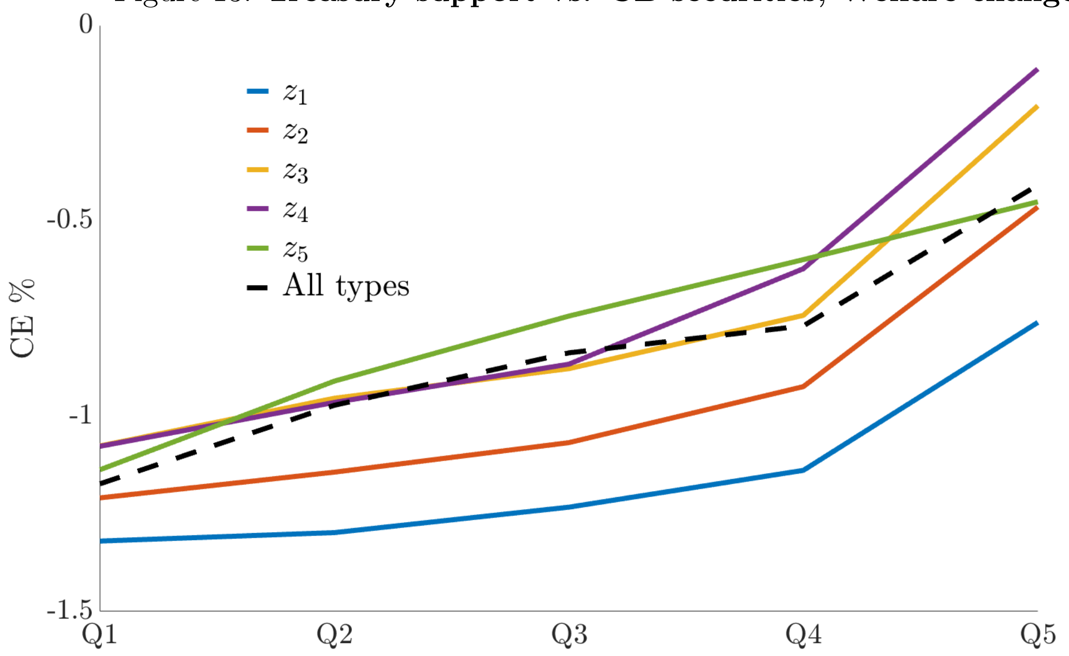
Lecture: Debt held by households, money and consumption are expressed in percentage deviation from reference values. Tax rate, real interest rate and inflation rate are expressed in % base points deviation from reference. CB profit and CB securities are expressed in value change from reference.

Figure 12: **Permanent QE**



A.3.2 Welfare analysis

Figure 13: **Treasury support vs. CB securities, Welfare change**



Lecture: CE by income quintile.